

Update of the Hadronic Vacuum Polarisation Contribution to the muon $g-2$

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Abstract

Precise data on $e^+e^- \rightarrow \text{hadrons}$ have recently become available and are used to compute the lowest-order hadronic vacuum polarisation contribution to the muon magnetic anomaly through dispersion relations. This is the case for the dominant $\pi^+\pi^-$ channel, but the most significant progress comes from the near completion of the *BABAR* program of measuring exclusive processes below 2 GeV with the initial-state radiation method which allows an efficient coverage of a large range of energies.. In this paper we briefly review the data treatment, the achieved improvements, and the result obtained for the full Standard Model prediction of the muon magnetic anomaly. The value obtained, $a_\mu^{\text{had LO}} = (692.6 \pm 3.3) \times 10^{-10}$ is 20% more precise than our last estimate in 2010. It deviates from the direct experimental determination by $(27.4 \pm 7.6) \times 10^{-10}$ (3.6σ). Perspectives for further improvement are discussed.

Keywords:

1. Hadronic vacuum polarisation to muon $g-2$ and e^+e^- data

The dominant part of the uncertainty in the Standard Model prediction for the muon magnetic anomaly $a_\mu = (g-2)/2$, where g is the gyromagnetic ratio equal to 2 at the lowest QED order, comes from the contribution of the lowest-order (LO) hadronic vacuum polarisation (HVP). The latter is computed through a dispersion relation using the measured cross sections for $e^+e^- \rightarrow \text{hadrons}$, as the relevant energy scale is too low for applying perturbative QCD. The HVP component is given by:

$$a_\mu^{\text{had LO}} = \frac{1}{4\pi^3} \int_{m_\pi^2}^{\infty} ds K(s) \sigma_{\text{hadrons}}^0(s), \quad (1)$$

where $K(s)$ is a QED kernel and $\sigma_{\text{hadrons}}^0(s)$ the bare cross section including final state radiation. Therefore progress on the HVP contribution is completely controlled by the availability of precise and reliable data on the hadronic annihilation cross sections.

Since our last update in 2010 [1] (see also Ref. [2]) new experimental data became available. In particular the *BABAR* collaboration has essentially completed

a program of precise measurements of exclusive cross sections for all the dominant channels of $e^+e^- \rightarrow \text{hadrons}$ from threshold to an energy of 3-5 GeV using the initial-state radiation (ISR) method. Also results are being produced at the VEPP-2000 facility in the 1-2 GeV range. In this paper we present our improved prediction using these new input data.

2. Data treatment

Our procedure for computing the dispersion relation has evolved with several new ideas to improve precision and reliability. At a time when the quality of e^+e^- data was limited we proposed in 1997 to use instead data from hadronic τ decays assuming CVC and taking into account isospin-breaking effects [3], taking advantage of the pure τ decay sample in the ALEPH experiment. Furthermore the relative normalization with respect to the τ leptonic decay was known very precisely from the measurement of the branching ratios. Through detailed QCD studies the τ hadronic spectral functions were shown to be well described by quark-hadron duality [4] so that one could propose and justify in 1998 to use perturbative QCD at energies as low

as the τ mass [5, 6]. With the availability of VEPP-2M more precise data a substantial update was published in 2003 [7, 8]. Unmeasured channels were estimated or bounded using isospin constraints. In 2010 a more detailed study of isospin breaking when using τ data was performed [9], yielding better agreement with e^+e^- data. Reliability was increased with improved statistical and systematic tools for the treatment and combination of the data from different experiments through the package HVPTools, and the *BABAR* $\pi^+\pi^-$ was included [10]. Finally the last global update using all the available e^+e^- data in 2010 produced the value [1]¹

$$a_\mu^{\text{had LO}} = (692.3 \pm 4.2) \times 10^{-10} \quad (2011) . \quad (2)$$

Experimental exclusive cross sections are integrated using Eq. (1) up to 1.8 GeV. In the present work 37 exclusive channels are included, as compared to only 22 in 2010. Thanks to the larger completeness of the data sample, only very few channels are now estimated with isospin constraints. In the energy range 1.8–3.7 GeV and above 5 GeV 4-loop perturbative QCD is used [11]. The contribution from the open charm region 3.7–5 GeV is again computed with experimental data. The narrow resonances J/ψ and $\psi(2S)$ Breit-Wigner line shapes are integrated using their currently best known parameters.

The integration of data points belonging to different experiments with their own data densities requires a careful treatment, especially concerning the correlated systematic uncertainties within the same experiment or between different experiments using the same tools. Quadratic interpolation of the data points is performed for each experiment and a local weighted average between the interpolations is computed in 1-MeV bins. Full covariance matrices are constructed between experiments and channels. Errors are propagated using pseudo-experiments (toys). When results from different experiments are locally inconsistent the error is rescaled according to the χ^2 value. At present, for the dominant $\pi^+\pi^-$ channel this is the major limiting factor for further improving the precision. Except for very few energy regions in a couple of channels the largest weight in the combination is obtained for the *BABAR* experiment.

3. The dominant $\pi^+\pi^-$ channel

The $\pi^+\pi^-$ channel dominates both the HVP contribution and its uncertainty. Recent experiments are generally limited by systematic uncertainties δ_{sy} . The main

¹When not explicitly noted, all a_μ^{had} values quoted in this paper are in units of 10^{-10} .

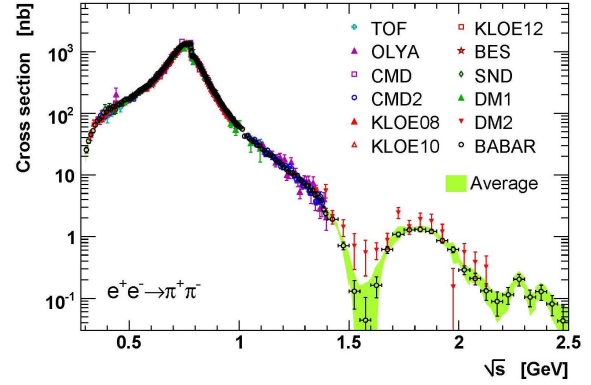


Figure 1: The measured bare cross sections for $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ with the combination band from threshold to 2.5 GeV. References are given in the text.

older contributors (references are given in Ref. [1] are CMD-2 ($\delta_{\text{sy}}=0.8\%$), SND (1.5%), KLOE-2008 (0.8%), KLOE-2010 (1.4%), *BABAR* (0.5%). For this update we included KLOE-2012 [12] (0.8%) and BESIII-2015 [13] (0.9%). Only *BABAR* covers the full mass range with high precision. Overall the combination in Fig.1 looks good. However a tension is observed between *BABAR* and KLOE in the ρ peak region (Fig.2), the other experiments falling in between and being consistent with both. It is important to compare experiments at the cross section level where they should agree, rather than using the integrated values where local discrepancies could artificially cancel and not be included in the quoted systematic uncertainty. Unhappily, the persisting discrepancy between *BABAR* and KLOE leads to an enlargement of the combined uncertainty

In spite of this problem, the progress in estimating the $\pi^+\pi^-$ contribution has been steady in the last decade. While the central value stayed within the quoted uncertainties, the total uncertainty dropped from 5.9 in 2003 to 2.9 in 2010, and now 2.5. More precisely the updated value from threshold to 1.8 GeV is $506.9 \pm 1.1_{\text{stat}} \pm 2.2_{\text{uncorsyst}} \pm 0.7_{\text{corsyst}}$, where the second (third) last uncertainty stands for systematic effects uncorrelated (correlated) with other channels, respectively. The correlation originates mainly from luminosity measurements and the VP correction.

Our last estimate [14] using τ decay data from ALEPH, OPAL, CLEO, and Belle, $516.2 \pm 2.9_{\text{exp}} \pm 2.2_{\text{IB}}$, where the second uncertainty arises from isospin-breaking (IB) corrections, is 2.2σ larger than the current e^+e^- -based value. The difference can be reduced by applying off-resonance $\gamma-\rho$ mixing corrections [15] with additional uncertainties. Because of the impressive progress of e^+e^- data, the τ input is now relatively

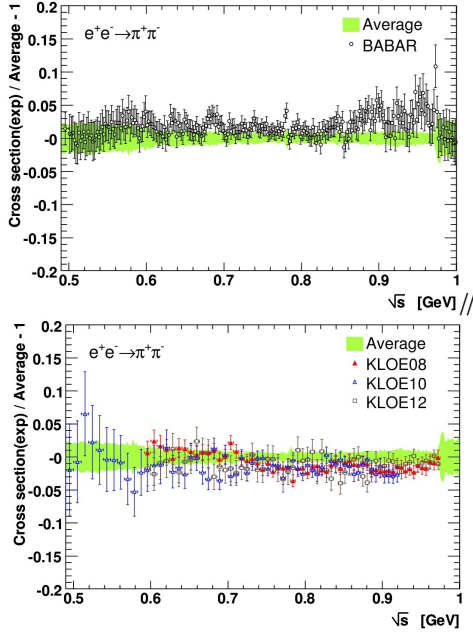


Figure 2: The ratio of the measured cross sections for $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ to the average for *BABAR* (top) and the different KLOE measurements (bottom).

less precise and less reliable due to the IB uncertainties. While the τ - e^+e^- comparison is interesting in its own right, it is safer now not to use it to evaluate precise HVP contributions.

4. The four-pion channels

Preliminary results from *BABAR* on $e^+e^- \rightarrow \pi^+\pi^-\pi^0(\gamma)$ have been presented [16]. As with other *BABAR* measurements using the initial-state-radiation (ISR) method with the ISR photon detected at large angle, the acceptance for the hadron system is large so that the final state structure can be studied and taken into account in the Monte Carlo generator, hence reducing significantly the systematic uncertainty on the acceptance. Data from some older experiments, both imprecise and inconsistent, are now discarded. As seen in Fig. 3 the *BABAR* results show a considerable improvement in precision.

The combined $\pi^+\pi^-\pi^0$ HVP contribution from threshold to 1.8 GeV to a_μ comes out to be $18.03 \pm 0.06_{\text{stat}} \pm 0.49_{\text{uncorsyst}} \pm 0.26_{\text{corsyst}}$ where the total uncertainty 0.55 is much reduced compared to the 2011 value (1.24). Recall the τ ALEPH estimate [14] based on the $\nu_\tau\pi^+\pi^+\pi^-$ and $\nu_\tau\pi^+3\pi^0$ decay modes, $21.02 \pm 1.16_{\text{exp}} \pm 0.40_{\text{IB}}$ which is 2.1σ larger, albeit much less precise.

For the $2\pi^+2\pi^-$ channel new data with the full *BABAR* sample were published [17] in 2012 with 5

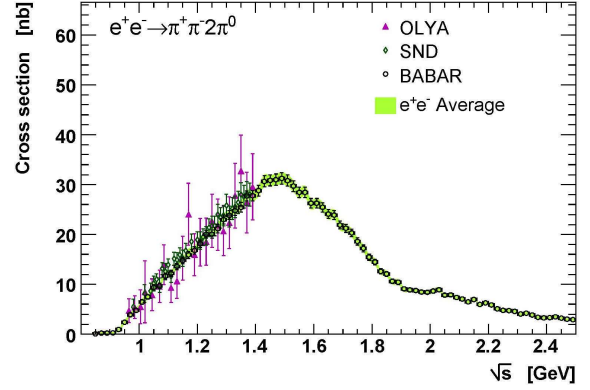


Figure 3: The measured bare cross sections for $e^+e^- \rightarrow \pi^+\pi^-\pi^0(\gamma)$ with the combination band from threshold to 2.5 GeV.

times more statistics and a smaller systematic uncertainty (2.4%). The resulting combined HVP contribution is now $13.70 \pm 0.03_{\text{stat}} \pm 0.28_{\text{uncorsyst}} \pm 0.13_{\text{corsyst}}$ with a reduced total uncertainty (0.31) compared to the 2011 value (0.53). The ALEPH τ estimate [14], $12.79 \pm 0.65_{\text{exp}} \pm 0.35_{\text{IB}}$, is consistent, but much less precise. Since the τ estimate for the two four-pion channels have some anticorrelation from the $\nu_\tau\pi^+3\pi^0$ mode through the isospin relations, it makes sense to combine the two channels. The τ value, 33.81 ± 1.53 , is then consistent with the corresponding e^+e^- value, 31.86 ± 0.64 , within 1.2σ . While there has been a steady progress in e^+e^- results over the last two decades, it is disappointing that similar advances have not occurred for the τ spectral functions. The situation is illustrated in Fig. 4.

5. The channels $K\bar{K}$

New data are available for the $K_s K_l$ channel: *BABAR* [18] detects both K_s and K_l from threshold to 2.2 GeV, while CMD-3 [19] measures only K_s in the ϕ resonance region. A good consistency is observed for the ϕ between the two experiments as well as with older ones (CMD-2 and SND). The cross section is given in Fig. 5.

The new $K_s K_l$ contribution to $a_\mu^{\text{had LO}}$ up to 1.8 GeV is $12.81 \pm 0.06_{\text{stat}} \pm 0.18_{\text{uncorsyst}} \pm 0.15_{\text{corsyst}}$ with a total uncertainty (0.24) reduced from the 2011 value (0.39).

Recent results from SND [20] at VEPP-2000 for the $K^+ K^-$ channel agree well with *BABAR* [21], while both show a discrepancy with the former SND results at VEPP-2M below 1.4 GeV beyond the quoted systematic uncertainty. The *BABAR* and the new SND data are displayed in Fig. 6.

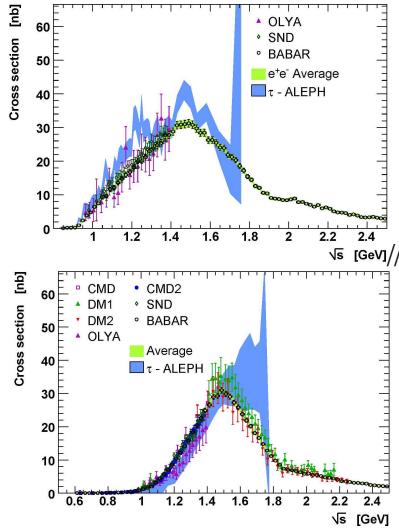


Figure 4: The measured bare cross sections for $e^+e^- \rightarrow \pi^+\pi^-2\pi^0(\gamma)$ (top) and $e^+e^- \rightarrow 2\pi^+2\pi^-(\gamma)$ (bottom) with the combination band from threshold to 2.5 GeV. The corresponding predictions from ALEPH τ spectral functions (darker band) are superimposed.

Some concern is arising on the $e^+e^- \rightarrow \phi \rightarrow K^+K^-$ cross section. The *BABAR* result has a systematic uncertainty of 0.7%, but it is higher by 5.1% (9.6%) with respect to CMD-2 (SND) with respective systematic uncertainties of 2.2% (7.1%). Including the *BABAR* data the new contribution to $a_\mu^{\text{had LO}}$ increases from 21.63 to 22.67 with a new uncertainty of 0.43. However a new preliminary result from CMD-3 [22] shows a huge increase ($\sim 11\%$) with respect to CMD-2, $\sim 5\%$ now above *BABAR*! It should be noted in this respect that the ISR method is more reliable than the scan technique for the detection of the slow charged kaons in the ϕ system because the kaons are boosted.

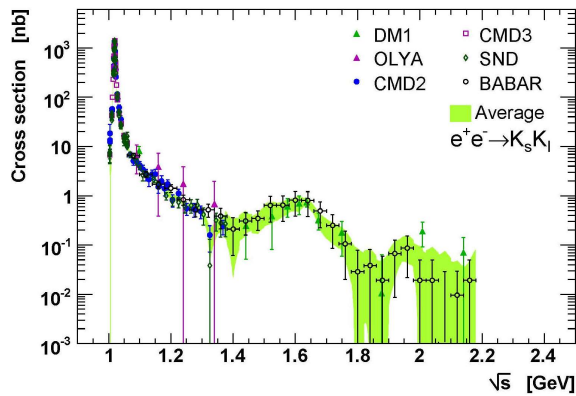


Figure 5: The measured bare cross sections for $e^+e^- \rightarrow K_s K_l(\gamma)$ with the combination band from threshold to 2.2 GeV.

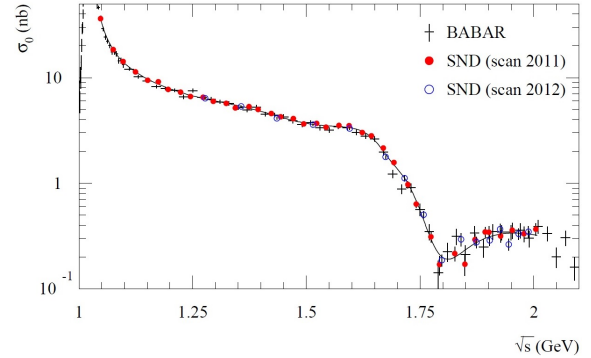


Figure 6: The measured bare cross sections for $e^+e^- \rightarrow K^+K^-(\gamma)$ from *BABAR* and SND at VEPP-2000.

6. The channels $K\bar{K} + \text{pions}$

For our previous analyses the available data on $e^+e^- \rightarrow K\bar{K} + n\text{pions}$ did not cover all the channels. Fortunately, it was possible to partially overcome this lack of information by using constraints based on the knowledge of final-state dynamics and isospin symmetry [7, 1]. This procedure is now becoming unnecessary due to the release of new results from the systematic measurements of exclusive processes by *BABAR*.

Together with previous measurements of $K_s K^\pm \pi^\mp$ and $K^+ K^- \pi^0$, data on the $K_s K_l \pi^0$ channel [23] now complete the picture for $n = 1$ (Fig. 7). The $K\bar{K}\pi$ final states being overwhelmingly dominated by $K^*(890)\bar{K} + cc$ below 1.8 GeV (with a small contribution from $\phi\pi^0$), it is not surprising that the isospin procedure works well. Indeed, the contribution from the sum of the measured channels is 2.45 ± 0.15 , in agreement (with similar precision) with the value 2.39 ± 0.16 using only $K_s K^\pm \pi^\mp$ data and isospin constraints.

For $n = 2$ many channels contribute, of which only two ($K^+ K^- \pi^+ \pi^-$ and $K^+ K^- 2\pi^0$) were known in 2010 from *BABAR*. Thus isospin constraints were used, but since the dynamics is more complicated here with final states $K^*(890)\bar{K}\pi + cc$, $K\bar{K}\rho$ and $\phi\pi\pi$, the systematic uncertainty had to be enlarged. New modes have now been added and older ones updated: $K^+ K^- \pi^+ \pi^-$ and $K^+ K^- 2\pi^0$ [24], $K_s K_l \pi^+ \pi^-$ and $K_s K_s \pi^+ \pi^-$ [18], and $K_s K_l 2\pi^0$ [23]. Apart for the $K_l K_l \pi^+ \pi^-$ channel which can be safely estimated using CP symmetry, all the cross sections have been measured. The only channel not yet released is $K_s K^\pm \pi^\pm \pi^0$, to appear shortly. The expected precision on the contribution from all $n = 2$ modes is 0.06, a large step from the previous isospin systematic uncertainty of 0.39 which is still kept until the final *BABAR* release.

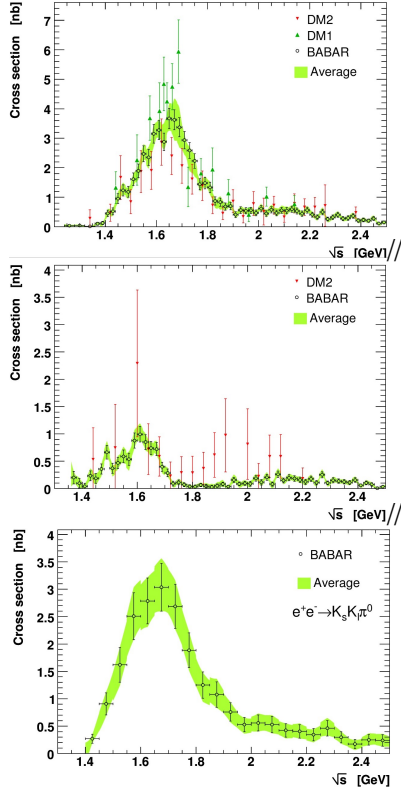


Figure 7: The measured bare cross sections for all $e^+e^- \rightarrow K\bar{K}(\gamma)$ channels from *BABAR* and older experiments.

7. Updated Standard Model prediction

Taking into account all the updated contributions our preliminary 2016 value for $a_\mu^{\text{had LO}}$ becomes

$$a_\mu^{\text{had LO}} = 692.6 \pm 1.2 \pm 2.6 \pm 1.6 \pm 0.1 \pm 0.3 \quad (2016). (3)$$

where the uncertainties are statistical, systematic from uncommon and common sources, from ψ , and QCD, respectively.

The central values for 2016 and 2011 Eq. (2) are in agreement. However the total uncertainty is significantly reduced from 4.2 to 3.3 (21%). Also the result shows the importance to take into account properly the systematic uncertainties which are correlated between channels for a given experiment and also between experiments.

Putting now together all the contributions to a_μ , $a_\mu^{\text{QED}} = 11658471.895 \pm 0.008$ [25], $a_\mu^{\text{EW}} = 15.4 \pm 0.1$ [26], $a_\mu^{\text{had LBL}} = 10.5 \pm 2.6$ [27], $a_\mu^{\text{had LO}} = 692.6 \pm 3.3$, $a_\mu^{\text{had NLO}} = -9.87 \pm 0.09$ [28], $a_\mu^{\text{had NNLO}} = 1.24 \pm 0.01$ [28], we obtain $a_\mu^{\text{SM prediction}} = 11659181.7 \pm 4.2$ to be compared to the direct measurement [29] $a_\mu^{\text{exp}} = 11659209.1 \pm 6.3$. Their difference, 27.4 ± 7.6 , remains

at the 3.6σ level, the reduction of the prediction uncertainty being compensated by the inclusion of the recently calculated NNLO hadronic contribution.

The nearly complete set of exclusive cross sections from *BABAR*, complemented by results from other experiments for some channels allows one to compute the total e^+e^- annihilation rate to hadrons $R(s)$, expressed in units of the point-like pair cross section. This is the most accurate determination to date which can be trusted up to 2 GeV. Clearly at larger \sqrt{s} values many more exclusive channels open up and inclusive R measurements are necessary. In this respect, as shown in Fig. 8 the newly published results from KEDR [30] between 1.84 and 3.05 GeV, complementing previous ones between 3.12 and 3.72 GeV [31] and the BES results [32], overlap nicely with our compilation and show excellent agreement with perturbative QCD, limiting serious deviations from local quark-hadron duality. The overall strongly damped oscillatory behaviour of $R(s)$ around the QCD prediction justifies using energy-averaged (global) quark-hadron duality [4] as a reliable tool to estimate dispersion integrals in the nearly-asymptotic regime.

8. Conclusion and perspectives

Using all available data on the $e^+e^- \rightarrow \text{hadrons}$ cross sections an update of the lowest-order hadronic vacuum polarisation to the muon magnetic anomaly is obtained with a relative precision of 0.5%: $a_\mu^{\text{had LO}} = (692.6 \pm 3.3) \times 10^{-10}$. The achieved uncertainty on this contribution is now reduced to about half the current uncertainty of the direct a_μ measurement. Thus the forthcoming programs at Fermilab [33] and JPARC [34], aiming at a precision four times smaller are therefore necessary to confirm if the present 3.6σ deviation is due to new physics beyond the Standard Model.

In order to match the precision of the future direct measurements, experimental progress is still needed to reduce further the uncertainty on $a_\mu^{\text{had LO}}$ from dispersion relations. Analyses for the $\pi^+\pi^-$ channel are underway with *BABAR* using a new independent method and CMD-3, where a systematic uncertainty of 0.3% looks reachable. In the 1-2 GeV range it will be important to continue to confront the *BABAR* and CMD-3/SND results. Independently, lattice calculations are also progressing, but they are not yet at a competitive level.

The precision of $a_\mu^{\text{had LO}}$ (3.3) is now getting close to the estimated systematic uncertainty on the hadronic LBL contribution $a_\mu^{\text{had LBL}}$ (2.6) which appears for the moment irreducible. Here only models have been used

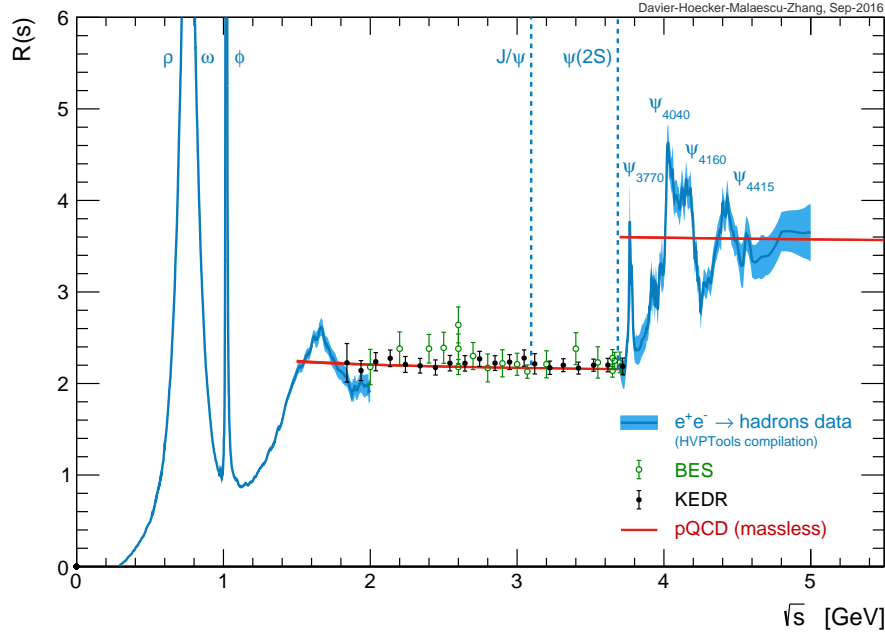


Figure 8: The total hadronic annihilation rate R as a function of \sqrt{s} . Inclusive measurements from BES [32] (and references therein) and KEDR [31, 30] are given as data points, while the sum of exclusive channels from this analysis is given by the narrow bands.

so far and lattice calculations are badly needed. However it should be pointed out that even if the LBL systematic uncertainty stays at the present level, the combined progress of e^+e^- data for the prediction and of the direct measurements would be sufficient to boost the present deviation (if persisting) to a level of 7σ , thereby allowing one to unambiguously claim a breakdown of the Standard Model.

I would like to thank A. Hoecker, B. Malaescu and Z. Zhang for the fruitful collaboration and our IHEP friends for organising a perfect τ workshop.

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